# **DNF-AF Selection Two-Way Relaying**

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**Abstract** Error propagation and noise propagation at the relay node would highly degrade system performance in two-way relay networks. In this paper, we introduce DNF–AF selection two-way relaying scheme which aims to avoid error propagation and mitigate noise propagation. If the relay successfully decodes the exclusive or (XOR) of the messages sent by the two transceivers, it applies denoise-and-forward (DNF). Otherwise, amplify-and-forward (AF) strategy will be utilized. In this way, decoding error propagation is avoided at the relay. Meanwhile, since the relay attempts to decode the XOR of the two messages instead of explicitly decoding the two messages, the larger usable range of XOR network coding can be obtained. As XOR network coding can avoid noise propagation, DNF–AF would mitigate noise propagation. In addition, bit error rate (BER) performance of DNF–AF selection scheme with BPSK modulation is theoretically analyzed in this paper. Numerical results verify that the proposed scheme has better BER performance than existing ones.

Keywords Two-way relay · DNF-AF selection · Error performance

## **1** Introduction

As a new way to explore spatial diversity in single antenna systems, cooperative diversity has been intensively investigated [1,2]. Decode-and-forward (DF) and amplify-and-forward (AF) are two basic modes in cooperative diversity. In DF mode, the relay decodes and re-encodes the received signal before forwarding the signal towards the destination. In AF mode, the relay amplifies and forwards the received signal to the destination. DF–AF selection relaying, where the relay switches between DF and AF adaptively, has also gained much attention [3–7].

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In order to redeem the loss in spectral efficiency due to the half-duplex constraint of the terminals, two-way relaying has been introduced. Two-way relaying via one half-duplex AF or DF relay has been investigated in [8]. Since then, different strategies such as AF, decodeand-superposition-forward (DSF) [9], joint DF (JDF), and denoise-and-forward (DNF) [10] are devised for two-way relay channel. AF two-way relaying strategy can be viewed as analog network coding for two-way relaying [11]. JDF and DNF make use of physical-layer network coding (PNC) in two-way relay channel. AF strategy maintains simplicity and cost-effectiveness while it introduces noise propagation. DSF, JDF and DNF can avoid noise propagation, but they may generate decoding error. AF two-way relay networks, DF two-way relay networks, and two-way relaying with DNF strategy have been intensively studied [12–18]. Adaptive two-way relaying, which extends the idea of DF–AF selection relaying to the two-way scenario, has been introduced in [19] and the outage probability has been studied. DF-JM scheme for two-way relay channel has been presented in [20].

Regarding the engineering application, [21] and [22] have investigated the utilization of two-way relaying in wireless sensor networks and LTE networks, respectively. When two-phase PNC protocol is employed in the two-way relaying, time synchronization is an important problem in practical applications [23]. The relay receives the mixture of two source signals whose timing offsets are usually different due to imperfect synchronization and the performance will decrease. Meanwhile, the AF schemes are typically limited by electronic amplifiers [24], and there are performance degradations in practical scenario. These physical limitations are universal in PNC two-way relaying. The discussions on these physical limitations can be found in works such as [23]. In the paper, we assume perfect time synchronization as well as perfect electronic amplifiers and focus on the theoretical aspects. Since the assumptions are universal in former related works (e.g., [19] and [20]), then the comparisons with former works are fair.

In this paper, a new two-way relaying scheme, termed DNF–AF selection scheme, is introduced. For the purpose of avoiding decoding error propagation and mitigating noise propagation, the relay node adaptively switches between DNF and AF according to its decoding state in the proposed scheme. Specifically, the main contributions can be summarized as follows:

- (1) A novel two-way relaying scheme is proposed in this paper. The relay attempts to decode the exclusive or (XOR) of the two messages instead of explicitly decoding the two messages. As it is easier to correctly decode the XOR of the two messages than correctly decode both messages, larger probability of using XOR network coding can be achieved. This leads to better mitigation of noise propagation at the relay. Detailed comparison between the proposed scheme and the adaptive two-way relaying [19] is presented in Sect. 3.
- (2) We analyze the error probability of the proposed DNF–AF selection scheme as well as the adaptive two-way relaying scheme. By comparison, we prove that the proposed DNF–AF selection scheme outperforms the adaptive two-way relaying scheme. Note that bit error rate (BER) analysis of the adaptive two-way relaying scheme is also novel.

This paper is organized as follows. Section 2 introduces the system model and explains DNF–AF selection scheme. In Sect. 3, we compare DNF–AF selection scheme with adaptive two-way relaying scheme investigated in [19]. Next, the BER performance of DNF–AF selection scheme is analyzed in Sect. 4. An upper bound has been derived. In addition, we also obtain an upper bound of BER for adaptive two-way relaying. Numerical results are presented and discussed in Sect. 5. Finally, we conclude the paper in Sect. 6.

*Notation:*  $\oplus$  denotes a bit-wise exclusive or (XOR) operation,  $x \sim C\mathcal{N}(\mu, \sigma^2)$  means that x is a circularly symmetric complex Gaussian random variable with mean  $\mu$  and variance  $\sigma^2.Q(\cdot)$  denotes the *Q*-function.

## 2 System Model and Description of DNF-AF Selection Two-Way Relaying

As shown in Fig. 1, we consider a two-way relay network consisting of two transceivers and a relay. Every node has only a single antenna and operates in half-duplex mode. There is no direct link between the two transceivers, and they exchange their information through the relay. We consider the flat-fading scenario. The complex reciprocal channel coefficients from the relay to transceivers  $S_1$  and  $S_2$ , denoted by  $h_1$  and  $h_2$  respectively, are assumed to be independent circularly symmetric complex Gaussian random variables with zero mean and unit variance, i.e.,  $h_i \sim C\mathcal{N}(0, \frac{1}{2})$  for i = 1, 2. We consider the 2-step scheme of two-way relaying in this paper. In the first step, both transceivers simultaneously transmit their data to the relay. The relay node then processes, reformats if necessary, and broadcasts the resulting signals to the transceivers in the second step. It is assumed that both  $h_1$  and  $h_2$  are available at  $S_1, S_2$ , and R.

Let  $\mathcal{M}_i$  be a constellation mapper used at  $S_i$  (i = 1, 2) in step 1. The transmitting symbols from  $S_1$  and  $S_2$  are given by  $x_1 = \mathcal{M}_1(m_1), x_2 = \mathcal{M}_2(m_2).m_1$  and  $m_2$  are digital source data per symbol from  $S_1$  and  $S_2$ , respectively.  $m_i \in \mathbb{Z}_{2^{k_i}} = \{0, 1, \dots, 2^{k_i} - 1\}$ , where  $k_i$  is the number of bits per symbol in  $\mathcal{M}_i$ . We assume that every constellation has unit energy.

The received signal at the relay node *R* in step 1 can be expressed as

$$y_r = \sqrt{P_1 h_1 x_1} + \sqrt{P_2 h_2 x_2} + w_r, \tag{1}$$

where  $P_i$  is the transmit power of  $S_i$ ,  $w_r \sim C\mathcal{N}(0, \sigma_r^2)$  is the AWGN at R.

In step 2, the relay uses the following to generate  $m_r$ .

$$m_{r} = \underset{\substack{x \oplus y \\ (x, y) \in \mathcal{A}}}{\operatorname{argmin}} \left| y_{r} - \sqrt{P_{1}}h_{1}\mathcal{M}_{1}(x) - \sqrt{P_{2}}h_{2}\mathcal{M}_{2}(y) \right|^{2}$$
(2)

with  $\mathcal{A} = (\mathbb{Z}_{2^{k_1}}, \mathbb{Z}_{2^{k_2}})$ .*m<sub>r</sub>* is the decoded XOR of the two transmitted messages. Specifically,

$$(a,b) = \underset{(x,y) \in \mathcal{A}}{\operatorname{argmin}} \left| y_r - \sqrt{P_1} h_1 \mathcal{M}_1(x) - \sqrt{P_2} h_2 \mathcal{M}_2(y) \right|^2$$
(3)

and

$$m_r = a \oplus b, \tag{4}$$

**Fig. 1** 2-step two way relaying. **a** step 1, **b** step 2



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i.e., the relay first jointly decodes the transmitted messages (*a* and *b* are the decoded results for  $m_1$  and  $m_2$ , respectively). Next, the relay gets the XOR of the two decoded results. If  $m_r = m_1 \oplus m_2$ , i.e., the relay could correctly decode the XOR of the two transmitted messages *R* sends  $x_r = \sqrt{P_r} \mathcal{M}_r(m_r)$  to both  $S_1$  and  $S_2$ , where  $P_r$  is the transmit power of the relay. Otherwise *R* amplifies  $y_r$  and broadcasts to  $S_1$  and  $S_2$ , i.e.,

$$x_r = \beta y_r,\tag{5}$$

where  $\beta = \sqrt{\frac{P_r}{P_1|h_1|^2 + P_2|h_2|^2 + \sigma_r^2}}$  is an amplification factor. That is to say, if the relay can decode the XOR correctly, the relay uses the DNF protocol; Otherwise, the relay utilizes the AF protocol.

*Remark* Observe that using DNF or AF is decided by the correctness of  $m_r$ . If  $a = m_1$  and  $b = m_2$  (i.e., both the two messages are decoded correctly),  $m_r$  is correct. However, both the two messages are decoded correctly is only a sufficient condition of the correctness of  $m_r$ . For BPSK, if  $a \neq m_1$  and  $b \neq m_2$  (i.e., both the two messages are not decoded correctly),  $m_r$  is also correct. This is the advantage of our proposed scheme: The relay attempts to decode the exclusive or (XOR) of the two messages instead of explicitly decoding the two messages. As it is easier to correctly decode the XOR of the two messages than correctly decode both messages, larger probability of using XOR network coding can be achieved. This leads to better mitigation of noise propagation at the relay.

Consequently, the received signal at  $S_i$  in step 2 is

$$y_i = h_i x_r + w_i, \tag{6}$$

where  $w_i \sim C\mathcal{N}(0, \sigma_i^2)$  denotes the AWGN at  $S_i$ . If  $m_r = m_1 \oplus m_2$ ,  $S_1$  decodes  $m_2$  using

$$\check{m}_2 = \underset{y \in \mathbb{Z}_{2^{k_2}}}{\operatorname{argmin}} \left| y_1 - \sqrt{P_r} h_1 \mathcal{M}_r(m_1 \oplus y) \right|.$$
(7)

Otherwise,  $S_1$  subtracts its own signal prior to decoding the signal sent by  $S_2$ , i.e.,  $S_1$  decodes  $m_2$  according to

$$\acute{m}_{2} = \underset{y \in \mathbb{Z}_{2^{k_{2}}}}{\operatorname{argmin}} \left| y_{1} - h_{1}\beta \left( \sqrt{P_{1}}h_{1}x_{1} + \sqrt{P_{2}}h_{2}\mathcal{M}_{2}(y) \right) \right|.$$
(8)

On the other hand,  $S_2$  decodes  $m_1$  similarly.

## 3 Comparison with Adaptive Two-Way Relaying

For adaptive two-way relaying considered in [19], in step 2, R employs the maximumlikelihood (ML) criterion to jointly decode<sup>1</sup> both messages from  $y_r$ , i.e.,

$$(\hat{m}_1, \hat{m}_2) = \underset{(x,y)\in\mathcal{A}}{\operatorname{argmin}} \left| y_r - \sqrt{P_1} h_1 \mathcal{M}_1(x) - \sqrt{P_2} h_2 \mathcal{M}_2(y) \right|^2.$$
(9)

If the relay could jointly decode  $m_1$  and  $m_2$  correctly, i.e.,  $(\hat{m}_1, \hat{m}_2) = (m_1, m_2)$ , R sends

$$x_r = \sqrt{P_r} \mathcal{M}_r(\hat{m}_1 \oplus \hat{m}_2)$$

<sup>&</sup>lt;sup>1</sup> We assume that the difference between  $P_1$  and  $P_2$  is not large, then successive interference cancelation (SIC) is not suitable.

to both  $S_1$  and  $S_2$ . For the hybrid case where only one message is successfully decoded, *R* will proceed to an intermediate state where the successfully decoded message is relayed through DF and the other is relayed through AF. Formally, when  $\hat{m}_i = m_i$ ,  $\hat{m}_j \neq m_j$  (*i*, *j*  $\in$ {1, 2}, *i*  $\neq$  *j*), *R* sends

$$x_{r} = \sqrt{P_{r,i}} x_{i} + \sqrt{\frac{P_{r,j}}{P_{j}|h_{j}|^{2} + \sigma_{r}^{2}}} (\sqrt{P_{j}}h_{j}x_{j} + w_{r}),$$

where  $P_{r,i}$  is the power used for DF,  $P_{r,j}$  is for AF, and  $P_{r,i} + P_{r,j} = P_r$ . When  $\hat{m}_1 \neq m_1, \hat{m}_2 \neq m_2$ , the relay uses (5) to generate the transmitting signal. The decoding method is similar as in the proposed scheme when  $(\hat{m}_1, \hat{m}_2) = (m_1, m_2)$  or  $\hat{m}_1 \neq m_1, \hat{m}_2 \neq m_2$ . For the hybrid case  $\hat{m}_1 = m_1$  and  $\hat{m}_2 \neq m_2$ ,  $S_1$  decodes  $m_2$  according to

$$\tilde{m}_{2} = \underset{y \in \mathbb{Z}_{2^{k_{2}}}}{\operatorname{argmin}} \left| y_{1} - h_{1} \left( \sqrt{P_{r,1}} x_{1} + \sqrt{\frac{P_{r,2}}{P_{2} |h_{2}|^{2} + \sigma_{r}^{2}}} \sqrt{P_{2}} h_{2} \mathcal{M}_{2}(y) \right) \right|$$
(10)

and  $S_2$  decodes  $m_1$  by using

$$\tilde{m}_{1} = \underset{y \in \mathbb{Z}_{2^{k_{1}}}}{\operatorname{argmin}} \left| y_{2} - h_{2} \left( \sqrt{P_{r,1}} \mathcal{M}_{1}(y) + \sqrt{\frac{P_{r,2}}{P_{2} |h_{2}|^{2} + \sigma_{r}^{2}}} \sqrt{P_{2}} h_{2} x_{2} \right) \right|.$$
(11)

The decoding method is similar when  $\hat{m}_1 \neq m_1$  and  $\hat{m}_2 = m_2$ .

The relay could switch adaptively between different schemes to avoid decoding error propagation in both DNF–AF selection scheme and adaptive two-way relaying. However, the scheme proposed in this paper is different from adaptive two-way relaying. In the proposed scheme, the relay tries to decode the XOR of the two messages instead of explicitly decoding the two messages.<sup>2</sup> Consequently, the proposed scheme has larger usable range of XOR network coding than adaptive relaying, and this will lead to better noise propagation mitigation. The BER performance of adaptive two-relaying has not been analyzed in [19]. In next section, we will analyze and compare the BER performance of DNF–AF selection scheme and adaptive two-relaying scheme.

#### 4 Error Performance Analysis

The probability of using DNF,  $P_{dnf}$ , is given by

$$P_{dnf} = \Pr\left\{m_r = m_1 \oplus m_2\right\}.$$
(12)

Furthermore, for BPSK, we have <sup>3</sup>

$$P_{dnf} = 1 - \Pr\left\{\hat{m}_1 = m_1, \hat{m}_2 \neq m_2\right\} - \Pr\left\{\hat{m}_1 \neq m_1, \hat{m}_2 = m_2\right\}.$$
(13)

Meanwhile, the error probability of BPSK for the ML detector can be bounded as [25]

$$\Pr\left\{\hat{m}_1 = m_1, \, \hat{m}_2 \neq m_2\right\} \le Q\left(\frac{|h_2|\sqrt{P_2}}{\sigma_r}\right)$$

 $<sup>^2</sup>$  In the proposed scheme, we donot proceed according to the correctness of the decoded pair. Instead, we proceed according to the correctness of the XOR of the two decoded messages.

<sup>&</sup>lt;sup>3</sup> Please refer to the Appendix for detailed derivation.

and

$$\Pr\left\{\hat{m}_1 \neq m_1, \hat{m}_2 = m_2\right\} \le Q\left(\frac{|h_1|\sqrt{P_1}}{\sigma_r}\right),$$

then

$$P_{dnf} \ge 1 - Q\left(\frac{|h_1|\sqrt{P_1}}{\sigma_r}\right) - Q\left(\frac{|h_2|\sqrt{P_2}}{\sigma_r}\right).$$
(14)

The probability of using AF can be given by

$$P_{af} = 1 - P_{dnf}.\tag{15}$$

Let  $P_{e,i}$  denote the BER at  $S_i$ . The end-to-end BER for the two-way relaying is defined as [15]

$$P_e = (P_{e,1} + P_{e,2})/2.$$
(16)

In the proposed scheme, we have

$$P_{e,i} = P_{dnf} \operatorname{Pr}\{\check{m}_{j} \neq m_{j}\} + P_{af} \operatorname{Pr}\{\check{m}_{j} \neq m_{j}\}$$

$$\stackrel{(a)}{=} \left(\operatorname{Pr}\{\check{m}_{j} \neq m_{j}\} - \operatorname{Pr}\{\check{m}_{j} \neq m_{j}\}\right) P_{dnf}$$

$$+ \operatorname{Pr}\{\check{m}_{j} \neq m_{j}\}$$
(17)

for  $i, j \in \{1, 2\}$  and  $i \neq j$ , where (a) holds since (15). Assume that BPSK modulation is used at the two transceivers. When DNF is applied at the relay, we have

$$\Pr\{\breve{m}_j \neq m_j\} = Q\left(\frac{2|h_i|\sqrt{P_r}}{\sqrt{2\sigma_i^2}}\right).$$
(18)

When the relay utilizes AF strategy,

$$\Pr\{\hat{m}_j \neq m_j\} = Q\left(\sqrt{\frac{2|h_i|^2 P_r P_j|h_j|^2}{|h_i|^2 P_r \sigma_r^2 + \sigma_i^2 \left(P_1|h_1|^2 + P_2|h_2|^2 + \sigma_r^2\right)}}\right).$$
 (19)

Comparing (18) and (19), we have  $Pr\{\check{m}_j \neq m_j\} < Pr\{\check{m}_j \neq m_j\}$ , i.e.,

$$\Pr\{\breve{m}_j \neq m_j\} - \Pr\{\breve{m}_j \neq m_j\} < 0.$$
<sup>(20)</sup>

Consequently, combing (14), (17) and (20), we get

$$P_{e} \leq \frac{1}{2} \left\{ \left[ \mathcal{Q}\left(\frac{|h_{1}|\sqrt{P_{1}}}{\sigma_{r}}\right) + \mathcal{Q}\left(\frac{|h_{2}|\sqrt{P_{2}}}{\sigma_{r}}\right) \right] \sum_{j=1}^{2} \sum_{i=1, i \neq j}^{2} \times \mathcal{Q}\left( \sqrt{\frac{2|h_{i}|^{2} P_{r} P_{j}|h_{j}|^{2}}{|h_{i}|^{2} P_{r} \sigma_{r}^{2} + \sigma_{i}^{2} \left(P_{1}|h_{1}|^{2} + P_{2}|h_{2}|^{2} + \sigma_{r}^{2}\right)} \right) + \left[ 1 - \mathcal{Q}\left(\frac{|h_{1}|\sqrt{P_{1}}}{\sigma_{r}}\right) - \mathcal{Q}\left(\frac{|h_{2}|\sqrt{P_{2}}}{\sigma_{r}}\right) \right] \times \sum_{j=1}^{2} \sum_{i=1, i \neq j}^{2} \mathcal{Q}\left(\frac{2|h_{i}|\sqrt{P_{r}}}{\sqrt{2\sigma_{i}^{2}}}\right) \right\} = P_{up,1}.$$
(21)

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Likewise, for adaptive two-way relaying, we can derive that

$$\begin{split} P_{e}^{'} &\leq \frac{1}{2} \left\{ \frac{1}{2} \left( \mathcal{Q} \left( \frac{|h_{1}\sqrt{P_{1}} + h_{2}\sqrt{P_{2}}|}{\sigma_{r}} \right) + \mathcal{Q} \left( \frac{|h_{1}\sqrt{P_{1}} - h_{2}\sqrt{P_{2}}|}{\sigma_{r}} \right) \right) \\ &\times \sum_{j=1}^{2} \sum_{i=1, i \neq j}^{2} \mathcal{Q} \left( \sqrt{\frac{2|h_{i}|^{2}P_{r}P_{j}^{2} + \sigma_{i}^{2} \left(P_{1}|h_{1}|^{2} + P_{2}|h_{2}|^{2} + \sigma_{r}^{2} \right)} \right) \\ &+ \left[ 1 - \mathcal{Q} \left( \frac{|h_{1}|\sqrt{P_{1}}}{\sigma_{r}} \right) - \mathcal{Q} \left( \frac{|h_{2}|\sqrt{P_{2}}}{\sigma_{r}} \right) \\ &- \frac{1}{2} \left( \mathcal{Q} \left( \frac{|h_{1}\sqrt{P_{1}} + h_{2}\sqrt{P_{2}}|}{\sigma_{r}} \right) + \mathcal{Q} \left( \frac{|h_{1}\sqrt{P_{1}} - h_{2}\sqrt{P_{2}}|}{\sigma_{r}} \right) \right) \right) \right] \\ &\times \sum_{j=1}^{2} \sum_{i=1, i \neq j}^{2} \mathcal{Q} \left( \frac{2|h_{i}|\sqrt{P_{r}}}{\sqrt{2\sigma_{i}^{2}}} \right) + \sum_{j=1}^{2} \sum_{i=1, i \neq j}^{2} \mathcal{Q} \left( \frac{|h_{j}|\sqrt{P_{j}}}{\sigma_{r}} \right) \\ &\times \left[ \mathcal{Q} \left( \sqrt{\frac{2|h_{i}|^{2}P_{r,j}\sigma_{r}^{2} + \sigma_{i}^{2} \left(P_{j}|h_{j}|^{2} + \sigma_{r}^{2}\right)} \right) \\ &+ \mathcal{Q} \left( \sqrt{\frac{2|h_{j}|^{2}P_{r,j}\sigma_{r}^{2} + \sigma_{j}^{2} \left(P_{j}|h_{j}|^{2} + \sigma_{r}^{2}\right)} \right) \right] \right\} \\ \coloneqq P_{up,2}. \end{split}$$

$$(22)$$

Observe that  $Q(\cdot)$  is decreasing function, we can obtain that  $P_{up,1} < P_{up,2}$ , i.e., the proposed scheme has lower upper bound than adaptive two-way relaying considered in [19]. Although the bounds are not the exact end-to-end BER, they give an important insight into the performance gap.

## **5** Numerical Results

In this section, BER performance of the proposed DNF–AF selection scheme is evaluated. In the simulations, the noise power at the relay and at the two transceivers is assumed to be  $\frac{1}{2}$ , i.e.,  $\sigma_r = \sigma_1 = \sigma_2 = \frac{\sqrt{2}}{2}$ . Using end-to-end BER as the design criterion, we compare DNF–AF selection scheme with two other schemes: adaptive two-way relaying [19] and DF-JM [20]. In the simulations, we set  $P_{r,1} = P_{r,2} = \frac{1}{2}P_r$  for hybrid case in adaptive two-way relaying.

We consider two scenarios in this paper.

- (1) Scenario 1: BPSK modulation is used at the two transceivers.
- (2) Scenario 2: BPSK is used at one transceiver and another transceiver utilizes QPSK.

*Remark* Scenario 1 is the symmetric case, for comprehensive comparisons, we also consider the asymmetric case, i.e., Scenario 2.

We first consider equal power allocation, i.e.,  $P_1 = P_2 = P_r = P/3$ .

Figure 2 plots the BER performance of DNF–AF selection scheme, adaptive two-way relaying and DF-JM scheme <sup>4</sup> in Scenario 1. As can be seen from the figure, both DNF–AF

<sup>&</sup>lt;sup>4</sup> With respect to the labeling of DF-JM, the optimal labeling scheme is applied in Scenario 1: symmetric scenario (Figs. 2 and 4) and the improved labeling map shown in Fig.1(c) of [19] is utilized for Scenario 2: asymmetric scenario (Figs. 3 and 5).



Fig. 2 BER performance for DNF-AF selection scheme, adaptive two-way relaying, and DF-JM in Scenario 1



Fig. 3 BER performance for DNF-AF selection scheme, adaptive two-way relaying, and DF-JM in Scenario 2

selection scheme and adaptive two-way relaying perform better than DF-JM. In fact, the relay could switch adaptively between different schemes to avoid decoding error propagation in both DNF-AF selection scheme and adaptive two-way relaying, and it is better than applying fixed scheme in DF-JM. On the other hand, DNF-AF selection outperforms adaptive twoway relaying. The reason is that DNF-AF selection has larger usable range of XOR network coding than adaptive relaying. When BPSK is used, XOR network coding can be used when  $(\hat{m}_1, \hat{m}_2) = (m_1, m_2)$  or  $\hat{m}_1 \neq m_1, \hat{m}_2 \neq m_2$  in DNF-AF selection. By contrast, XOR network coding can only be used when  $(\hat{m}_1, \hat{m}_2) = (m_1, m_2)$  in adaptive two-way relaying. When  $\hat{m}_1 \neq m_1$  and  $\hat{m}_2 \neq m_2$ , adaptive two-way relaying utilizes AF. As XOR network



Fig. 4 BER performance with different power allocations in Scenario 1, the total power is P = 20 dBW



Fig. 5 BER performance with different power allocations in Scenario 2, the total power is P = 20 dBW

coding can avoid noise propagation, this results in better noise propagation control in the proposed scheme. Then the BER performance of the proposed scheme is better.

Figure 3 shows the BER performance of DNF–AF selection scheme, adaptive two-way relaying and DF-JM scheme in Scenario 2. In this scenario, dummy zeros padding [26] is used. From Fig. 3, we can notice that the proposed scheme can also achieve the best BER performance in asymmetric case.

In the following, we investigate unequal power allocations. Define the power allocation coefficient  $\alpha = P_r/P$  to represent the power allocation between the source and relay. In the simulation, we assume equal source power, i.e.,  $P_1 = P_2$ . Then,  $P_1 = P_2 = \frac{1-\alpha}{2}P$  and  $P_r = \alpha P$ .

Figure 4 illustrates the BER performance under different  $\alpha$  in the symmetric scenario. We can observe that the proposed DNF-AF scheme outperforms the DF-JM scheme and adaptive



Fig. 6 BER performance of the proposed DNF–AF selection scheme with general power allocations among the two sources and relay in Scenario 1, the total power is P = 20 dBW

scheme under all  $\alpha$ . Especially when  $\alpha = 0.4, 0.5, 0.6, 0.7$ , the advantages are obvious. In addition, the power allocation has strong influence on the BER performance of all three schemes. For our proposed scheme, the BER first increases and then decreases when we increase  $\alpha$ . The reason is that when  $\alpha$  is small, i.e.,  $P_1 = P_2$  is large, the relay can correctly decode the XOR of the two source messages with high probability in the first time-slot. So the BER is approximated by (18), and then the BER decreases with the increase of  $P_r$  (i.e., increase of  $\alpha$ ). However, when the  $\alpha$  is larger than some value,  $P_1 = P_2$  is not large enough to support the correctly decoding of the XOR at the relay. Then the BER is approximated by (19). When we increase  $\alpha$ , the difference between  $P_r$  and  $P_1 = P_2$  increases.<sup>5</sup> Thus, the BER increases according to (19). Furthermore, from the figure and the analysis, we can guess that an optimal  $\alpha$  may exist. Figure 5 depicts the BER performance under different  $\alpha$  in the asymmetric scenario. We can see that the DNF–AF selection scheme also achieves the best performance in this scenario.

In the above simulations, the source powers are equal, i.e.,  $P_1 = P_2$ . Next, we consider the general power allocations among the two sources and relay. Let  $P_1 = \beta P$ ,  $P_2 = \gamma P$ , and then  $P_r = (1 - \beta - \gamma)P$ . Figure 6 shows the BER of the DNF-AF selection scheme with general power allocations (For better understanding, we plot the BER performance when  $\beta = 0.1$  and  $\beta = 0.3$  as Fig. 7 and plot the BER performance when  $\gamma = 0.1$  as Fig. 8, they are sub-figures of Fig. 6).<sup>6</sup> We can see that when  $\beta$  is small (e.g. 0.1, 0.15, 0.2, 0.25, 0.3), the BER first decreases, and then increases while we decrease  $\gamma$ . When  $\gamma$  is small (e.g. 0.1, 0.15, 0.2, 0.25, 0.3, 0.35), the BER first decreases, and then increases when we increase  $\beta$ .

<sup>&</sup>lt;sup>5</sup> The difference between  $P_r$  and  $P_1 = P_2$  is 0 when  $\alpha = \frac{1}{3}$ . If  $\alpha > \frac{1}{3}$ , the increase of  $\alpha$  will result in the increase of the difference.

<sup>&</sup>lt;sup>6</sup> In the simulations, we first set  $\beta = 0.1, 0.15, 0.2, 0.25, 0.3, \dots, 0.8$ , and next set  $\gamma = 0.1, \dots, 1 - \beta - 0.1$  to guarantee that all power allocations are positive.



**Fig. 7** Sub-figure of Fig. 6 when  $\beta = 0.1$  and  $\beta = 0.3$ , respectively



Fig. 8 Sub-figure of Fig. 6 when  $\gamma = 0.1$ 

## 6 Conclusion

We have proposed DNF–AF selection scheme for two-way relay channels in this paper. By switching between DNF and AF according to the decoding state adaptively, the relay node could avoid decoding error and mitigate noise propagation. Through analysis of the BER performance, we have proved that DNF–AF selection scheme has lower upper bound of BER than adaptive two-way relaying. Finally, simulation results demonstrate that DNF–AF selection scheme has better BER performance than both adaptive two-way relaying and DF-JM scheme. Furthermore, simulation results show that the power allocation has strong impact on the performance, and we will investigate the optimal power allocation in future works.

[Derivation of (13)] The decoding at the relay is according to (2). For the error analysis, it is equivalent to

$$(\hat{m}_1, \hat{m}_2) = \operatorname*{argmin}_{(x,y)\in\mathcal{A}} \left| y_r - \sqrt{P_1} h_1 \mathcal{M}_1(x) - \sqrt{P_2} h_2 \mathcal{M}_2(y) \right|^2$$

combined with

$$m_r = \hat{m}_1 \oplus \hat{m}_2.$$

We have only 4 cases for comparing  $(\hat{m_1}, \hat{m_2})$  and  $(\hat{m_1}, \hat{m_2})$ :

1.  $m_1 = \hat{m}_1$  and  $m_2 = \hat{m}_2$ 2.  $m_1 \neq \hat{m}_1$  and  $m_2 \neq \hat{m}_2$ 3.  $m_1 = \hat{m}_1$  and  $m_2 \neq \hat{m}_2$ 4.  $m_1 \neq \hat{m}_1$  and  $m_2 = \hat{m}_2$ .

Hence, we can derive

$$\Pr \{m_1 = \hat{m}_1, m_2 = \hat{m}_2\} + \Pr \{m_1 \neq \hat{m}_1, m_2 \neq \hat{m}_2\} + \Pr \{\hat{m}_1 = m_1, \hat{m}_2 \neq m_2\} + \Pr \{\hat{m}_1 \neq m_1, \hat{m}_2 = m_2\} = 1.$$
(23)

For BPSK, we get

$$\Pr\{m_r = m_1 \oplus m_2\} = \Pr\{m_1 = \hat{m}_1, m_2 = \hat{m}_2\} + \Pr\{m_1 \neq \hat{m}_1, m_2 \neq \hat{m}_2\}.$$
(24)

Combining (12), (23) and (24), we arrive at (13).

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### References

- Sendonaris, A., Erkip, E., & Aazhang, B. (2003). User cooperation diversity-part I: System description. *IEEE Transactions on Communications*, 51(11), 1927–1938.
- Laneman, J. N., Tse, D. N. C., & Wornell, G. W. (2004). Cooperative diversity in wireless networks: Efficient protocols and outage behavior. *IEEE Transactions on Information Theory*, 51(12), 3062–3080.
- Zhao, B., & Valenti, M. (2003). Some new adaptive protocols for the wireless relay channel. In Proceedings
  of the annual allerton conference on communication control and computing, Monticello, IL.
- Souryal, M. R., & Vojcic, B. R. (2006). Performance of amplify-and-forward and decode-and-forward relaying in Rayleigh fading with turbo codes. In *Proceedings of IEEE ICASSP*, Toulouse, France.
- Li, Y., & Vucetic, B. (2008). On the performance of a simple adaptive relaying protocol for wireless relay networks. In *Proceedings of IEEE VTC-Spring'08*, Singapore.
- Su, W., & Liu, X. (2010). On optimum selection relaying protocols in cooperative wireless networks. *IEEE Transactions on Communications*, 58(1), 52–57.
- Zhang, T., Chen, W., & Cao, Z. (2012). Opportunistic DF-AF selection relaying with optimal relay selection in Nakagami-m fading environments. In Proceedings of the 1st IEEE international conference on communications in China (IEEE ICCC'12), Beijing, China.
- Rankov, B., & Wittneben, A. (2005). Spectral efficient signaling for half-duplex relay channels. In Proceedings of asilomar conference on signals, systems, computers, Monterey, CA.
- Chen, M., & Yener, A. (2010). Power allocation for F/TDMA multiuser two-way relay networks. *IEEE Transactions on Wireless Communications*, 9(2), 546–551.
- Popovski, P., & Yomo, H. (2007). Physical network coding in two-way wireless relay channels. In Proceedings of IEEE ICC'07, Scotland.
- Nazer, B., & Gastpar, M. (2011). Reliable physical layer network coding. *Proceedings of the IEEE*, 99(3), 438–460.

- 12. Li, C., Yang, L., & Shi, Y. (2010). An asymptotically optimal cooperative relay scheme for two-way relaying protocol. *IEEE Signal Processing Letters*, *17*(2), 145–148.
- Ngo, H. Q., Quek, T. Q. S., & Shin, H. (2010). Amplify-and-forward two-way relay networks: error exponents and resource allocation. *IEEE Transactions on Communications*, 58(9), 2653–2666.
- Yang, J., Fan, P., Duong, T. Q., & Lei, X. (2011). Exact performance of two-way AF relaying in Nakagamim fading environment. *IEEE Transactions on Wireless Communications*, 10(3), 980–987.
- Ju, M., & Kim, I.-M. (2010). Error performance analysis of BPSK modulation in physical-layer networkcoded bidirectional relay networks. *IEEE Transactions on Communications*, 58(10), 2770–2775.
- Lo, E. S., & Letaief, K. B. (2011). Design and outage performance analysis of relay-assisted two-way wireless communications. *IEEE Transactions on Communications*, 59(4), 1163–1174.
- Chen, W., Hanzo, L., & Cao, Z. (2011). Network coded modulation for two-way relaying. In *Proceedings* of *IEEE WCNC'11*.
- Koike-Akino, T., Popovski, P., & Yomo, H. (2009). Optimized constellations for two-way wireless relaying with physical network coding. *IEEE Journal on Selected Areas in Communications*, 27(5), 773–787.
- 19. Li, Q., Ting, S. H., Pandharipande, A., & Han, Y. (2009). Adaptive two-way relaying and outage analysis. *IEEE Transactions on Wireless Communications*, 8(6), 3288–3299.
- Chen, Z., Liu, H., & Wang, W. (2010). A novel decoding-and-forward scheme with joint modulation for two-way relay channel. *IEEE Communications Letters*, 14(12), 1149–1151.
- Chen, Z., Kuehne, A., Klein, A., Loch, A., Hollick, M., & Widmer, J. (2012) Two-way relaying for multiple applications in wireless sensor networks. In *Proceedings of 2012 international ITG workshop* on smart antennas (WSA'12).
- Hamdoun, H., Loskot, P., O'Farrell, T., & He, J. (2011). Practical network coding for two way relay channels in LTE networks. In *Proceedings of VTC Spring'11*.
- Zhang, S., Liew, S.-C., & LAM, P. P. (2006). On the synchronization of physical-layer network coding. In Proceedings of 2006 IEEE information theory workshop.
- 24. Cripps, S. C. (2006). RF power amplifiers for wireless communications (2nd ed.). Artech House Inc.
- 25. Verdu, S. (1998). Multiuser detection. Cambridge: Cambridge University Press.
- 26. Zhao, J., Kuhn, M., Wittneben, A., & Bauch, G. (2010). Asymmetric data rate transmission in two-way relaying systems with network coding. In *Proceedings of IEEE ICC'10*, Cape Town, South Africa.



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